The Prediction of Brazilian Exports Using Bayesian Forecasting

Helio S. Migon

Universidade Federal do Rio de Janeiro
C.P. 68507, Rio de Janeiro, RJ 21945-970, Brazil
migon@pep.ufrj.br

Abstract
An illustration of Bayesian forecasting to predict the Brazilian industrial exports is described in this paper. This application can be viewed as an elaborated pedagogical exercice to forecast an economic time series based on data for a period ot time when the dolar exchange rate was controlled by the Brazilian government. A system of simultaneous equations is developed for the supply and demand of industrial goods. An equation representing the 'value of exports', which is simply a function of the real exchange rate and the world income is obtained, using a logarithmic specification. The dynamic behavior is introduced on an 'ad-hoc' basis using the general adaptive expectation hypothesis. Although the emphasis is on the applications, a brief description of the dynamic Bayesian forecasting method is also presented.

Keywords: Dynamic Bayesian Forecasting. Transfer Response Model. General Adaptive Expectation. Simultaneous Equations Model.

1 Introduction

The aim of this paper is to present models for short term predictions of the value of the Brazilian industrial exports using time varying parameters. Moreover, we will also investigate the shape of the response function in relation to variations on the real exchange rate.

Bayesian forecasting methodology is nowadays well documented although it is recognized that there are a few real case studies presented in the literature. In this paper we are concerned in providing a simple but interesting illustration of dynamic Bayesian Forecasting to predict an economic time series. The illustration presented here is part of a large econometric study developed in the midle 80's for the Brazilian economy.

Following *Goldstein and Khan (1978)* a simple supply and demand model is developed. Using a logarithmic transformation, the simultaneous equations model can be greatly simplified, yielding a simple regression explaining the value of the industrial exports.

To simplify the prediction system a dynamic linear growth model *(Pole, West and Harrison - 1994)* is used in place of some variables intentionally omitted from the model, such as the domestic capacity and the level of the world economy. The omission of those variables is expected to be satisfactorily handled through a time varying system.

The dynamic behavior of the forecasting model is introduced through a general adaptive expectation hypothesis *(Leamer and Stern - 1970)*. One of the derived models, a Bayesian transfer response model, is a particular example of a very broad class of non-linear and non-normal Bayesian model introduced by *Migon (1984)* and *West, Harrison and Migon (1985)*. Many special features of the Bayesian approach to forecasting are stressed in the application, including variance modeling and estimation, subjective intervention and the use of discount factors.

The paper is organized as follows: Section 2 introduces the theory underlying the models developed for forecasting purposes, including the dynamic specifications. In Section 3, the models are framed as dynamic Bayesian models and inferential aspects of the non-linear models are discussed. Finally, in Section 4, the main findings of the models developed are presented.

2 Modeling Industrialized Brazilian Exports

The basic premise of the model developed to forecast the exports of industrial goods is the following: the model is based on an equilibrium model that enables a static relationship between the value of industrial goods exports and the real exchange rate. The dynamics of the model are introduced through a mechanism of general adaptive expectation.

Since the relationship between prices and quantities is needed in the equilibrium, it is worth pointing out that for forecast purposes, it will be sufficient to work with the reduced form of the model.

2.1 Supply and Demand Equations - an equilibrium model

The amount of goods exported depends on profit expectations and conditions of the internal market. Exporters must decide if they prefer to produce for the internal or external market. In these conditions the *supply* function depends upon the price in american dollar for the exporters (p), the exchange rate (r), i.e. the rate 'cruzeiro' per dollar divided by a Brazilian price index, the gap between the potential and real output of the economy (u) and the American price index (π).

To model the decision of the agents importing Brazilian goods it will be assumed that the *demand* is a function of the export price (p) and the world income (z). Using a log-linear specification for price and amount of exports, the simultaneous equations model in reduced form is:

$$ln(y^e) = \alpha_0^1 + \alpha_1^1 ln(z) + \alpha_2^1 ln(u) + \gamma_1 ln(r) ln(\frac{p}{\pi}) = \alpha_0^2 + \alpha_1^2 ln(z) + \alpha_2^2 ln(u) + \gamma_2 ln(r) \qquad (1)$$

where: y^e represents the quantum index in the market equilibrium. Adding the above equations results:

$$ln(\frac{p}{\pi} y^e) = ln(y^v) = \alpha_0 + \alpha_1 ln(z) + \alpha_2 ln(u) + \gamma ln(r) \qquad (2)$$

The logarithm of the *value of exports*, denoted by y_t^v, is a function of the *level of the world economy* (z_t), the *gap between the potential and real output* (u_t) of the exporter economy, and the *real exchange rate* (r_t).

To keep the model simple the variables z_t and u_t will be replaced by a *stochastic trend* component since from the Brazilian government point of view the exchange rate is its only short term control variable, at least for the period of time involved in this study. It follows, therefore, that:

$$ln(y_t^v) = \mu_t + \gamma ln(r_t) \qquad (3)$$

where μ_t is a linear function of the time encompassing information on z and u.

Economic agents, however, do not react instantaneously to variations on this control variable. To improve the model we need to consider the agents' decision effects along a time horizon.

2.2 Dynamic Specification and the Proposed Model

The dynamics of the system in study can be explored via the general adaptive expectations theory leading to the dynamic model specification

$$y_t = \gamma_0 x_t + \gamma_1 x_{t-1} + \cdots + \gamma_p x_{t-p} + \delta_1 y_{t-1} + \cdots + \delta_q y_{t-q} \qquad (4)$$

relating the variables y_t and x_t, with suitable choice of p and q. Alternative approaches include the mechanism of partial adjustment (see for details *Leamer and Stern* (1970)).

Combining the static relationship that emerges from the simultaneous model (3) and the general adaptive expectations hypothesis (4) yields the model:

$$ln(y_t^v) = \mu_t + E_t + \nu_t \qquad (5)$$

where ν_t is a random disturbance and $E_t = \gamma_0 ln(r_t) + \gamma_1 ln(r_{t-1}) + \cdots + \gamma_p ln(r_{t-p}) + \delta_1 ln(y_{t-1}^v) + \cdots + \delta_q ln(y_{t-q}^v)$ is the response function. As can be seen the dynamic model is composed of three basic terms. The first is a stochastic trend, representing information on omitted variables, the second is the response function to changes in the real exchange rate and the last one a random component. The inclusion of a seasonal component can be considered if it is worth.

The variables z_t and u_t were intentionally omitted from the conceptual model in order to keep it simple and sophisticated but apt to forecast the value of exports, conditionally to the exchange rate. This is justified, for the period of time involved in this study, as far as the exchange rate was used to be controlled by the Brazilian economic authority.

A very special case of (5) is obtained when $\gamma_i = \lambda^i \gamma_0$ and $\delta_i = 0,\ i = 1, \cdots, q$,

$$ln(y_t^v) = \mu_t + \frac{\gamma_0}{1 - \lambda L} r_t + \nu_t \qquad (6)$$

where L is the lag operator.

In the next section, model (5) will be written in the Bayesian forecast nomenclature as a dynamic linear model and the special case (6) will be phrased as a dynamic non-linear model. Aspects of the non-linear inferential procedure are discussed and some particular cases of the dynamic non-linear model are also presented.

3 Bayesian Dynamic Forecasting Models

A general class of Bayesian forecasting models is characterized by a Gaussian observational distribution and a linear regression equation linking the mean of the observational distribution with the so called state parameters. Extensions for non-linear and non-normal Bayesian models are easily introduced.

The general dynamic model is defined by the quadruple $\{F(.), G(.), \sigma^2, W\}_t$, where $F(.)$ and $G(.)$ are general smooth functions defining the mean η_t and the state parameters evolution and σ^2 and W represent the observational and evolution variances respectively. Some special models in this broad class will be discussed in the application.

3.1 Dynamic Linear Models

The model in equation (5) can be expressed as a general dynamic linear model, with $F_t = (F_1, F_2, F_3)'_t$ and $G_t = diag(G_1, G_2, G_3)$, where:

$$F_1 = (1, 0)', \quad G_1 = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}; \quad F_2 = (1, 0, \cdots, 0)' \quad G_2 = \begin{pmatrix} 0 & I_{s-1} \\ 1 & 0 \end{pmatrix};$$

$$F_3 = (ln(r_t), ln(r_{t-1}), \cdots, ln(r_{t-p}), ln(y_{t-1}^v), \cdots, ln(y_{t-q}^v)), \quad G_3 = I_{p+q}$$

This is obtained superposing three blocks component: trend, seasonally and a dynamic regression. Theoretical and practical aspects of these models can be found in West & Harrison (1997) and in Pole, West & Harrisson (1994).

3.2 Dynamic Non-Linear Models: first order transfer response

Our model is a first order transfer response plus trend as defined from equation (6):

$$ln(y^v) = \eta_t + \nu_t$$

where: $\eta_t = \mu_t + E_t$ and $\nu_t \sim N(0, \sigma_t^2)$, with $E_t = \gamma_t/(1-\lambda_t)\ ln(r_t)$. Note that the dynamics behind this formulation depends upon the parameters λ_t and γ_t making the model non-linear on them. To evolve the parameters through time we define the following equations:

$$\begin{pmatrix} \mu \\ \beta \\ E \\ \lambda \\ \gamma \end{pmatrix}_t = \begin{pmatrix} 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & \lambda_{t-1} & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} \mu \\ \beta \\ E \\ \lambda \\ \gamma \end{pmatrix}_{t-1} + \gamma_t \cdot \begin{pmatrix} 0 \\ 0 \\ ln(r_t) \\ 0 \\ 0 \end{pmatrix} + \omega_t \qquad (11)$$

where: $\omega_t \sim N(0, W_t)$, with $W_t = diag(w_\mu, w_\beta, w_E, w_\lambda, w_\gamma)_t$. All the parameters involved are time varying making the model very flexible and general.

In many applications of Bayesian forecasting $W_t = W$ and $\sigma_t^2 = a^{-1}$, $\forall t$. These models are called constant models. Some special constant models include:

i) a *static model* when the variances $w_\lambda = w_\gamma = 0$. The corresponding parameters are not time varying and the Bayesian inferential procedure provides only the sequential estimation of the transfer response parameters;

ii) a *linear growth plus first order transfer response* with $w_\mu = 0$ and $w_\beta = 0$ corresponds to a model in the first difference of the variables. That is:

$$\triangle ln(y_t^v) = \beta + \triangle E_t + \triangle \nu_t, \quad \text{since: } \beta_t = \beta, \quad \forall t \qquad (12)$$

where $\triangle$ is the difference operator and the evolution equation is defined as:

$$E_t = \lambda_t E_{t-1} + \gamma_t ln(r_t)$$

$$\begin{pmatrix} \lambda \\ \gamma \end{pmatrix}_t = \begin{pmatrix} \lambda \\ \gamma \end{pmatrix}_{t-1} + \begin{pmatrix} \omega_\lambda \\ \omega_\gamma \end{pmatrix}_t \qquad (13)$$

Another interesting case occurs when only $w_\beta = 0$ – it is a model in the second order difference of the input and output series. Finally, notice that when $\omega_{E_t} \sim N(0, w_E)$ we have a stochastic transfer responses model.

To complete the model specification the joint prior distribution for the mean η_t and the scale factor a_t is defined as a normal-gamma distribution:

$$(a_t|D_{t-1}) \sim \Gamma(\frac{n_t}{2}, \frac{s_t}{2})(\eta_t|D_{t-1}, a_t) \sim N(f_t, \frac{q_t}{a_t}) \qquad (14)$$

where: f_t and q_t are the prior mean and variance for η_t given D_{t-1} and the scale factor a_t, with: $D_t = [(ln(y_t^v), ln(r_t)), \cdots, (ln(y_1^v), ln(r_1)), D_0]$. The marginal prior distribution of the scale factor a_t given D_{t-1} is a gamma distribution with parameters n_t and s_t, as usual in Bayesian conjugate analysis. The prior expected value of a_t is $\frac{n_t}{s_t}$, where n_t represents the degrees of freedom, as can be seen in *West and Harrison (1997).*

The state parameter θ_t is defined as: $\theta_t = (\mu, \beta, E, \lambda, \gamma)_t'$ and at time t the posterior information available for $(\theta_t|D_t)$ is specified only partially in terms of first and second moments: $(\theta_t|D_t) \sim [m_t, C_t]$. The Bayesian inferential procedure operates sequentially through time and will provide the updating of information.

3.3 Bayesian Inference Procedure

To implement the inferential procedure we suppose that $ln(y_t^v)$ is normally distributed with mean η_t and variance $\sigma_t^2 = a_t^{-1}\eta_t$. The sequential procedure involves two operations: evolution and updating, and processes the information as follows:

i) Evolution to time t is performed through (11) to give the prior distribution for the state parameters conditional on a_t, D_{t-1}. On the other hand, the scale factor a_t evolves in such a way that it keeps its expected value and increases its uncertainty. Then the posterior distribution of the scale factor is given by:

$$(a_t|D_{t-1}) \sim \Gamma(\delta_a \frac{n_{t-1}}{2}, \delta_a \frac{s_{t-1}}{2})$$

where $0 < \delta_a \leq 1$ is a discount factor. The concept of discount factor is used to increase the uncertainty as time goes on. For evolution purposes we treat the state parameters as two independent blocks (linear growth and transfer response) keeping their covariance unaltered (*see Migon, 1984*). It is advisable to use different discount factors. For the trend component the discount factor is often in the interval (.90, .95) and we should keep the discount factor for the transfer function block very close to 1.

ii) The information obtained on observing y_t^v is used to update the distributions of $(\eta_t|a_t, D_t)$ and $(a_t|D_t)$, as usual in the normal conjugate analysis.

For the non-linear models the procedure follows *Migon (1984)* and is similar to West, Harrison and Migon (1985) for dynamic generalized linear models. The guide relationship linking θ_t and η_t is used for updating the state parameters. The information that comes from y_t is used to revise the η_t distribution and it is fed back to the θ_t through its relation with η_t.

4 The data and the main findings

The period of the data used in this application, Jun./79 up to Dec./84, is characterized by a deep recession in the world trade and a large devaluation of the Brazilian exchange rate, making it attractive to illustrate the capabilities of the models developed. *The value of exports of Brazilian industrial goods*, in millions of dollars, *the exchange rate* and *the retail price index*, denoted IPA/OG for industrialized goods, were obtained from The Bulletin of the Brazilian Central Bank.

Some forecasting results developed with the previous models are discussed in this section. It is useful to start with the dynamic linear models and progress to the non-linear case.

One practical aspect of the Bayesian forecasting model is the simplicity with which a practitioner can intervene to incorporate subjective information. It is well recognized that the information available to a forecaster is not restricted to past data, but may include information about events or changes affecting the series. In cases in which it is perceived that some events had not been accounted for, action is taken to feed back information to keep up the forecast performance.

In Figure 1, the on line fitting, posterior mean ($E[\mu_t|D_t]$), obtained using a simple linear growth model with and without subjective intervention, is shown. The uncertainty associated to the level is strongly increased, at the intervention time, improving considerably the predictive performance. The mean absolute error (MAD) decreases from .12 to .09 and the logarithm of the predictive likelihood (LPL) raises from 35 to 47 supporting the need of intervention. It is worth pointing out that the interventions were necessary to cope with change in the growth in Feb./82 and Jun./82 as it is evident from Figure 1.

Similar conclusions can be seen from Figure 2 where the smoothed mean of the level ($E[\mu_t|D_T]$) is plotted. These results, similar to classical data fitting, permit to observe more clearly the intervention effect as far as many randomness were ruled out.

Examining the residuals of these fits two alternatives are conceivable: to include a seasonal component or some of the omitted variable. In this study, the exchange rate (r_t) is the candidate suggested by the theoretical model developed in Section 2. In fact seasonality is not supported by the data (MAD = .12 and LPL = 37). On the other hand, including the control variable - the exchange rate - improves definitely the results. The dynamic regression model was assessed varying p and q. The selected model (p = q = 1) gave a MAD = .096 and a LPL = 41.4 which compares with the first order transfer response model (MAD = .086 and LPL = 49.5) unfavorably.

The best prediction results were obtained with the first order transfer response model, which is simple and sophisticated. In the graph below, Figure 3, one can appreciate the predictive capacity of the model and in Figure 4, the smoothed effect

Fig 1 : On-Line Fitting of Exports
Linear Grouth Model
without and with intervention

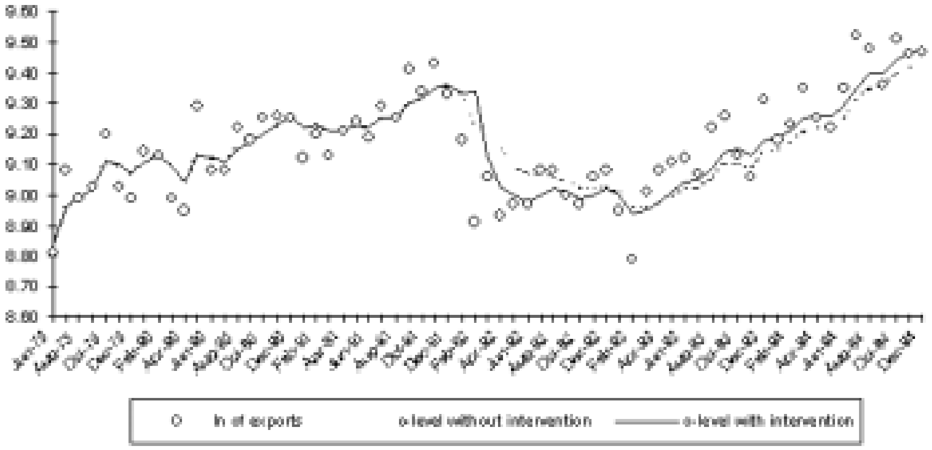
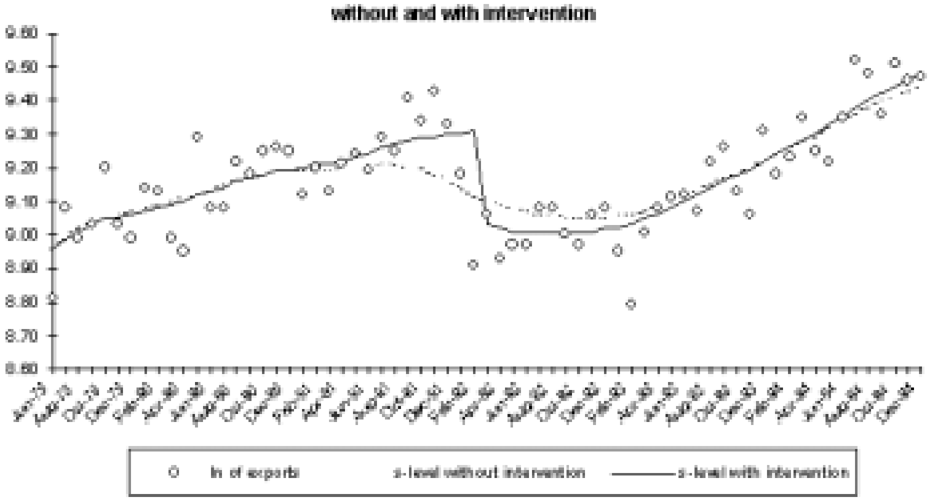


Fig 2 : Smoothed Fitting of Exports
Linear Grouth Model
without and with intervention



of the exchange rate.

In the remainder of this Section some practical aspects evolved in the implementation of non-linear dynamic models will be presented, besides many useful facilities

Fig 3 : On Line Fitting of Exports
Transfer Function Model
without and with intervention

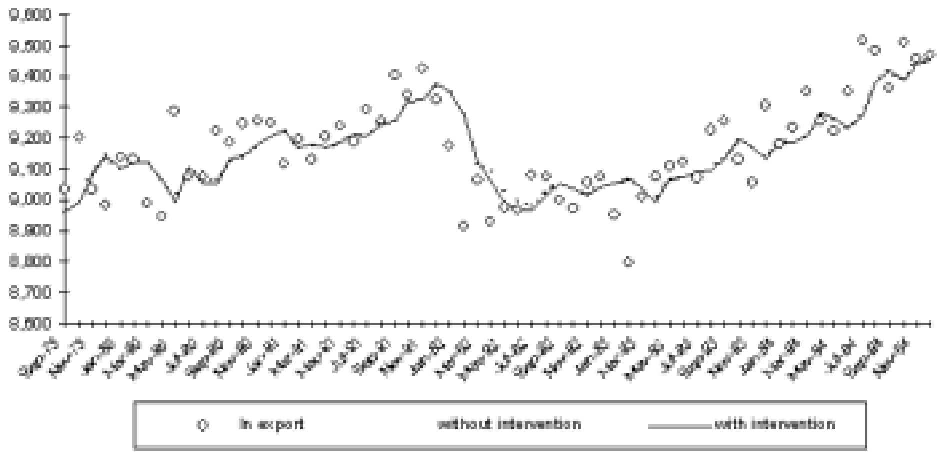
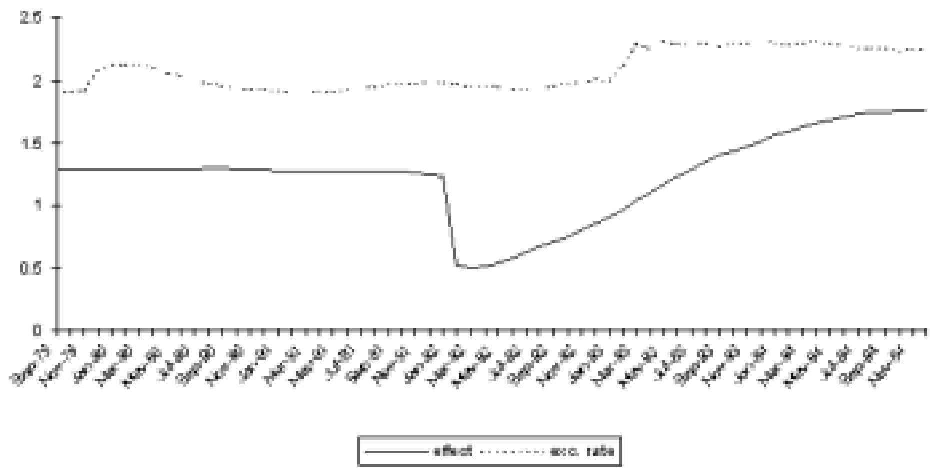


Fig 4 : Exchange Rate Effect



due to the Bayesian forecasting methodology, like discount factors, interventions, variance law etc.

To initialize the non-linear Bayesian forecasting system it is necessary to supply some prior information. The non-linear components need to be well located. The level of the process was supposed to be 8 units in a log scale with an uncertainty roughly described by probability limits of 95% given by $(0, 16)$. For the transfer response part, we started with mean value of $(\lambda, \gamma) = (.5, 0)$ and variance in correspondence with the 95% probability limits of $(-2, 2)$ *and* $(-20, 20)$, respectively. These again correspond to very vague prior information.

Instead of using a stochastic term in the evolution equation, as stated in Section 3, we have used the conceptually simple idea of discount factors. Since we are interested in short term forecasting we believe that the observations made in a distant past - for example 13 periods of time or more - are worth much less than the most recent past. The discount factor to the trend block was set assuming that in $N = 13$ periods of time the information content decays to half of its initial value. This roughly corresponds to a discount factor of .95, using the working relationship, $(3N-1)/(3N+1)$, given by *Harrison and Johnston (1983).* On the other hand, the discount factor for the transfer response block was kept at one, which signifies that the distribution of these parameters will be sequentially revised, but they are not varying through time. The observational variance, in the log scale, was assumed to be proportional to the process level, that is to say: $\sigma_t^2 = a_t^{-1}\eta_t$, where a_t was sequentially estimated from the data.

This is an example of stochastic trend model with a first order transfer response superposed on it. This express the effects of variation of the exchange rate on the exports of industrialized goods.

In this study we also will illustrate how to handle interventions, since in December/81 an international economic crises began and in Feb./83 a maxidevaluation of the Brazilian currency (cruzeiro) was implemented. At the time of interventions the discount factor associated with the level was decreased to .10 in order to incorporate the 'subjective information' that the user is uncertain about the future.

For example, the intervention in Dec./81 could be representing the analysts knowledge about the performance of the international economy. Remember that this variable was omitted to keep the model parsimonious. In Feb./83 a similar situation occurs in respect to the maxidevaluation of the Brazilian currency (cruzeiro), about 25% at one time. Although the response to exchange rate is one component of the model, it was necessary to intervene. In the graph it is very clear that the observation made at Feb./83 is an outlier. The exporters may have anticipated (some sort of inside information) and delayed, intentionally, their exports. If no action is taken at that time a complete misleading trend is obtained from that point on. The level will be badly underestimated and the predictive capacity of the model is greatly impaired. It is worth pointing out that the discrepant observation was considered as a missing data since the information it provides is not correct.

Examining the one step ahead residuals, it is evident that no structure remains in the residuals. The discounted R^2 *(Harrison and Johnston, 1983)* was .57 and all the parameters, as given by the posterior in Dec./84, were statistically significant:

	mean	t-statistic
μ	8.56	32.3
λ	0.49	14.9
γ	0.17	3.4

Tab. 1: The posterior mean of the parameters and the t-statistic.

Finally the long term elasticity $\gamma/(1-\lambda)$ is .333 and the response function has the following weights:

lag's	1	2	3	4	5
weights	.17	.08	.04	.02	.01

Tab. 2: The response function pattern.

indicating that there is a quite strong instantaneous response to the currency devaluation followed by some minor effects. In order to take care of the strong variability in the data a different aggregation could be tried, although it is not at all clear which is the best to be used for short time forecast.

5 Concluding Remarks

In this paper a simple but nice illustration of dynamic Bayesian forecasting for predicting an economic time series is discussed. Some practical aspects evolved in the implementation of non-linear dynamic models were presented besides the useful facilities due to the Bayesian forecasting methodology.

The best dynamic regression models corresponding to $p = q = 1$ gave a MAD=0.096 and a LPL=41.4 which compares with the first order transfer response (MAD = .086 and LPL = 49.5), our prefered model. To assess the effect of intervention a simple time series model, the linear growth, is introduced. The MAD drops from .12 to .09 and LPL raises from 35 to 47 demonstrating the effectiviness of intervene.

Finally it is worth pointing out that this example is part of large econometric model developed for the Bazilian economy in the middle of the 80's and continuously improved since then.

References

Goldstein, M and Khan, M. S (1978), *The Supply and Demand for Exports: a simultaneous approach*, The Review of Economics and Statistics, 275-285.

Harrison, P.J and Johnston, F. R (1984), *Discount Weight Regression*, Journal of Operational Research Society, 35 (1984), 10, 923-932.

Leamer, E. E and Stern, R. M (1970), *Quantitative International Economics*, Boston, Allyn and Bacon.

Migon, H. S (1984) - *An Approach to Non-Linear Bayesian Forecasting with Applications*. PhD Thesis, University of Warwick, England.

Migon H. S and Harrison, P. J (1985), *An Application of Non-Linear Bayesian Forecasting to TV Advertasing*, in *Bayesian Statistics II*, eds. J. M Bernardo, M. H DeGroot, D. V Lindley and A. F. M Smith, Amsterdam: North Holland.

Pole, A., West, M and Harrison, P. J (1994), *Applied Bayesian Forecasting and Time Series Analysis*, Chapman-Hall.

West, M and Harrison, P. J (1997), *Bayesian Forecasting and Dynamic Models*, 2nd Edition. Springer Verlag.

West, M, Harrison, P. J and Migon, H. S (1985), *Dynamic Generalized Linear Model and Bayesian Forecasting*. Journal of the American Statistical Association, 80, 73-97.